Complexity Days
Program, titles and abstracts of contributions

December 13th and 14th, 2023

1 Program

Wednesday 13th

- **14:00–15:00,** Sophie Laplante (invited talk)
- **15:30–16:30,** Martin Grohe (invited talk)
- **16:45–17:15,** Romain Pechoux
- **17:15–17:40,** Baptiste Chanus
- **17:40–18:00,** Djamel Amir

Thursday, 14th

- **9:00–10:00,** Jakob Nordström (invited talk)
- **10:30–11:00,** Yann Strozecki
- **11:00–11:30,** Andrei Romaschenko
- **11:30–12:00,** Maher Mallem
- **14:00–15:00,** Sébastien Tavenas (invited talk)
- **15:30–16:00,** Julien Grange
- **16:00–16:20,** Manon Blanc
- **16:20–16:40,** Ulysse Léchine
- **16:40–17:00,** Closing

2 Titles and abstracts of talks

- Martin Grohe (RWTH Aachen)

  Title: The Descriptive Complexity of Graph Neural Networks

  Abstract: Graph neural networks (GNNs) are deep learning models for graph data that play a key role in machine learning on graphs. A GNN describes a distributed algorithm carrying out local computations at the vertices of the input graph. Typically, the parameters governing this algorithm are acquired through data-driven learning processes.

  After introducing the basic model, in this talk we will focus on the expressiveness of GNNs: which functions on graphs or their vertices can be computed by GNNs? An intriguing and nontrivial facet of this inquiry is that GNNs are "analog" computation models operating on and with real numbers. Nevertheless, we can give precise characterisations of the expressiveness of GNNs in terms of Boolean circuits and logic, that is, computation models of classical (descriptive) complexity theory.

- Sophie Laplante (Université Paris Cité, IRIF)

  Titre: Certificate games

  Abstract: Consider the following two-player game for a Boolean function. The players are given inputs with different function values and are asked to output some index i such that $x_i \neq y_i$. (This is the well-known Karchmer Wigderson relation.) We study the setting where the players cannot communicate. We define Certificate Game complexity CG(f) to be the multiplicative inverse of the probability of winning this game, for the best strategy, in the worst case.

  We study Certificate game complexity, in its public-coin, shared entanglement, and non-signaling variants, and relate them to Certificate complexity and randomized query, and other measures of boolean function complexity.

  This is joint work with Sourav Chakraborty, Anna Gal, Rajat Mittal and Anupa Sunny.

- Jakob Nordström (University of Copenhagen and Lund University)

  Titre: Complexity Theory for Real-World Computation

  Abstract: Boolean satisfiability (SAT) and other NP-complete problems have been intensely studied for over 50 years, and a rich and mathematically sophisticated body of work has been developed exploring the consequences of the hypothesis that these problems are not solvable in polynomial time or are even exponentially hard. However, practitioners mostly think of SAT as an easy problem, and there are very efficient applied algorithms that can often solve formulas with even millions of variables in close to linear time. It is hard to think of a starker disconnect between theory and practice.

In this talk, we will discuss how computational complexity tools can be used to analyse algorithms that are widely used in practice. In addition to proving unconditional exponential lower bounds on running time, this study can also lead to ideas for how to improve such algorithms, and how to enhance them to produce machine-verifiable certificates of correctness for their computations.

- Sébastien Tavenas (CNRS, Université de Chambery, LAMA)

  Title : Superpolynomial Lower Bounds Against Low-Depth Algebraic Circuits : An overview

  Abstract : Every multivariate polynomial $P(x_1, ..., x_n)$ can be written as a sum of monomials, i.e. a sum of products of variables and field constants. In general, the size of such an expression is the number of monomials that have a non-zero coefficient in $\mathbf{P}$. What happens if we add another layer of complexity, and consider sums of products of sums (of variables and field constants) expressions? Now, it becomes unclear how to prove that a given polynomial $P(x_1, ..., x_n)$ does not have small expressions. We will discuss the background behind this question, the relations with standard Boolean complexity and some basic results from this area. Finally we will present the last results on these questions.

- Djamel Amir (Université Nancy, LORIA)

  Title : The expressive power of low descriptive complexity invariants

  Abstract : We study the descriptive complexity of topological invariants of compact Polish spaces. Many usual invariants are known to have non-Borel complexity, we are interested in the expressiveness of low complexity invariants. We fully characterize the Π^0_1 invariants, which are all about connectedness properties, and we obtain several results about Σ^0_2 invariants and their ability to differentiate between spaces. Notably, we show that they are sufficient for distinguishing finite topological graphs.

- Manon Blanc (École Polytechnique, LIX)

  Title: Characterisations of polynomial-time and -space complexity classes over the reals

  Abstract: Many recent works have studied how analogue models work, compared to classical digital ones. By "analogue" models of computation, we mean computing over continuous quantities, while "digital" models work on discrete structures, like bits. It led to a broader use of Ordinary Differential Equation (ODE) in computability theory. From this point of view, the field of implicit complexity has also been widely studied and developed. We show here, using arguments from computable analysis, that we can algebraically characterise PTIME and PSPACE for functions over the reals, using discrete ODEs and more precisely the so-called Linear Length ODEs schemata.

- Baptiste Chanus (Université Paris-Nord, LIPN)

  Title : Characterization of complexity classes using categorical logic

  Abstract: Fagin's theorem states that the class NP may be characterized as the class of problems which may be expressed in existential second order logic. It started the field of descriptive complexity, the development of which has shown a deep link between logic and complexity. In an attempt to understand this link from the categorical viewpoint, Damiano Mazza introduced an approach, based on Boolean lextensive categories and categorical logic, in which several standard complexity classes (recursivably enumerable languages, P, NP...) may be characterized in a natural way. In this talk, we will carry this approach further, showing how to characterize several more standard complexity classes: the polynomial hierarchy, PSPACE, the logarithmic time hierarchy and NL.

- Julien Grange (Université Paris-Est, LACL).

  Title : Specification and Automatic Verification of Computational Reductions

  Asbtract : We look at the question, given two algorithmic problems, of determining whether a given candidate reduction is indeed a reduction between these two problems, for varying classes of reductions and problems. Although we observe, unsurprisingly, that the validation problem is undecidable even for very restricted choices, we identify several decidable cases that are relevant in the context of teaching the basics of complexity theory. We propose an easy-to-use graphical specification mechanism for computational reductions and show the decidability of the validation problem for natural subclasses within this framework, thus allowing in this context automatic verification of reductions input by students.

  Joint work with Fabian Vehlken, Nils Vortmeier and Thomas Zeume.

- Ulysse Léchine (Université Paris-Nord, LIPN).

  Title : Coming up with new solutions independently : A simple to state but hard to solve combinatorial problem with links to time bounded Kolmogorov complexity

  Abstract : We propose the following game : we have a list L of 4 numbers ranging from 1 to 100 and two players A and B who each get half of that list. A and B can both output one number in [1;100] which may depend on their half of the list. They collectively win if at least one of them outputs a number which was not originally in the list L. They may agree on a strategy before hand but may not communicate once they've revceived their half of the list. Is there a winning strategy ? This (apparently) simple problem and its generaliztion to mutiple players and multiple numbers reveal quite the interesting mathematical analysis and has strong links with time bounded Kolmogorov Complexity.

- Maher Mallem,Sorbonne Université, (Sorbonne Université, LIP6).

  Title: Scheduling meets parameterized complexity.

  Abstract: In scheduling many problems are known to be strongly NP-hard, even seemingly basic ones. Upon reaching NP-hardness this quickly, it becomes difficult to establish anything deeper about these problems within the scope of classical complexity theory.

  To answer this, parameterized complexity theory gives additional tools for a refined analysis of such hard scheduling problems. Although parameterized complexity has been successfully used in graph theory for forty years, it has only recently drawn increasing interest in scheduling. Such problems typically involve parameters which are specific to the field.

  In this talk we will give several recent uses of the parameterized complexity framework in scheduling. In particular we will focus on parallel machine scheduling problems with release dates, deadlines and precedence delays. Two parameters will be considered: the pathwidth μ, which in the context of scheduling is the maximum number of overlapping job time windows at any given time, and the maximum delay value l_{max} which can appear in the precedence constraints.

- Romain Pechoux (Université de Lorraine, LORIA)

  Title : Implicit characterization of the class of Basic Feasible Functionals

  Abstract : In this talk, we will discuss recent characterizations of the class of Basic Feasible Functionals, a complexity class extending the notion of polynomial time to second order. We will show that it can be characterized by a programming language in an implicit way (i.e., without explicit bounds) and in a tractable way (i.e., decidable in polynomial time).

- Andrei Romaschenko, Montpellier

  Title: Triple mutual information: combinatorial and communication perspectives."

  Abstract in English : Kolmogorov complexity of a string x denoted C(x) is the length of the shortest program that produces x (given no input). Similarly, C(x,y) is the length of the shortest program that produces the pair (x,y). The linear combination C(x) + C(y) - C(x,y) is by definition the mutual information between x and y. This quantity is a natural measure of correlation between x and y; it is one of the most classical notions of algorithmic information theory.

  In this talk we will discuss a similar quantity of mutual information for a triple of strings. We will see that this notion helps to clarify some combinatorial arguments. We will discuss the usage of the triple mutual information in several results on communication complexity (classical and more recent ones).

  Résumé en Français : La complexité de Kolmogorov d'un mot x notée C(x) est la longueur du programme le plus court qui produit x (sans entrée). De même, C(x,y) est la longueur du programme le plus court qui produit la paire (x,y). La combinaison linéaire C(x) + C(y) - C(x,y) est par définition l'information mutuelle entre x et y. Cette quantité est une mesure naturelle de corrélation entre x et y ; c'est l'une des notions les plus classiques de la théorie algorithmique de l'information.

Dans cet exposé, nous discuterons d'une quantité similaire d'information mutuelle définie pour un triplet de mots. Nous verrons que cette notion permet de clarifier certains arguments combinatoires. Nous discuterons de l'utilisation de la triple information mutuelle dans plusieurs résultats sur la complexité de la communication (classiques et plus récents).

- Yann Strozecki (Université de Versailles, David)

  Title: Enumeration complexity and regularization

  Abstract : Enumeration, that is listing objects, is a common task in computer science, whether it is for counting them, finding an optimal solution, or building a library of interesting objects. The complexity measures for enumeration must take into account that the number of generated object is huge with regards to the input size. The most popular measure is the delay, the time between the production of two solutions. A more flexible measure is incremental time, which gauges the time required to generate k solutions. When an algorithm generates k solutions in time kf(n), we say that the algorithm has amortized delay f(n).

  It is common to turn a polynomial amortized delay into a worst case polynomial delay by using a large buffer to store solutions and regularize the enumeration. Unfortunately, this technique uses potentially exponential space.

  We present an alternative method, geometric regularization, which allows regularization of an algorithm in polynomial space without additional time cost. This method is essentially optimal, since it works even when the space used, the amortized delay and the number of solutions generated by the algorithm undergoing regularization are unknown.